Some Math 360 sample questions for review

Incidence, Euclid's and Hilbert's axioms will be provided if needed.

Q 1 Give an example of a finite incidence geometry with at most six points which does not satisfy Playfair's Postulate (that every line has exactly one parallel through each point not on that line)

Q 2 State the definition of a statement being independent of a set of axioms and give an example of a statement which is independant of the incidence axioms.

Q 3 Let ABC be an equilateral triangle. Identify the rigid motion which is the composition of rotation by $2\pi/3$ around C followed by translation from A to B.

Q 4 The set of symmetries of Figure A is the same as that for Figure B- true or false. (where Figures A and B are wallpaper patterns or frieze patterns, for example.)

Q 5 Which of the betweenness axioms hold for the integer lattice geometry, where points are of the form (m, n) where m and n are integers and lines are of the form $p(y - m) = q(x - n)$ for m, n, p and q integers?

Q 6 Find a circle p if possible such that the image of the circle of radius 2 centered at the origin is the line y=12 under inversion through p.

Q 7 Prove that Euclid's parallel postulate implies Playfair's parallel postulate.

Q 8 Prove that if two rigid motions agree on three non-collinear points, then they agree everywhere.

Q 9 Prove that in incidence geometry, if l and m are distinct lines which are not parallel, then they have a unique point in common.

Q 10 Show that if AB is a segment which is perpedicular to a line m, then the composition $\rho_m \circ \tau_{AB} = \rho_n$ where n is a line parallel to m.

Q 11 Give an example of two non-trivial glide reflections whose product is a rotation by $\pi/2$.

Q 12 State the definition of two points being symmetric with respect to a circle p and show that the only points that are symmetric to themselves with respect to p are the points on the circle.

Q 13 Show that $\rho_m \circ \rho_p \circ \rho_l = \rho_l \circ \rho_p \circ \rho_m$ if distinct lines l, m and p have a common perpendicular.

Q 14 Give an example of two rotations in the plane which proves that rotations do not always commute.

Q 15 a) Describe two different models M_1 and M_2 for incidence geometry, both which have exactly four points.
b) Give an example of a statement which is independent of the incidence axioms which is illustrated as being independent by being true in your M_1 and false in your M_2 from part a.

Q 16 a) State the definition of a rigid motion m of the Euclidean plane.

b) The definition of a reflection ρ_l across the line l is that a point P is sent to a point P' if l is the perpendicular bisector of the segment PP'. Prove that a reflection across l is a rigid motion of the Euclidean plane.

Q 17 Let A be the point $(0,0)$, let B be the point $(0,2)$ and let C be the point $(2,2)$. Consider the rigid motion $R_{B,\,\pi/2} \circ \tau_{AC}$, which is rotation counterclockwise around B by $\pi/2$ following translation by the vector $< 2, 2 >$.
a) What is the image of the point $(0,0)$ under this composition?
b) Identify the rigid motion $R_{B,\,\pi/2} \circ \tau_{AC}$.

Q 18 Let the square $ABCD$ have corners at $A = (0,0)$, $B = (1,0)$, $C = (1,1)$, and $D = (0,1)$.
Identify the rigid motion $R_{D,\,\pi} \circ \gamma_{CD}$ where $R_{D,\,\pi}$ is rotation by π around D and γ_{CD} is a glide reflection along the segment CD.

Q 19 Sketch an example of a Frieze pattern (a figure with a single axis of discrete translational symmetry, generally a strip-type pattern) with a set of symmetries that contains a glide reflection along a horizontal line and a rotation by π, but does not contain reflectional symmetry across a horizontal line. Indicate centers of rotational symmetry with a $\circ$ symbol.

Q 20 Let A be the point (0,0), B be the point (2,0) and C be the point (0,2). Consider the rigid motion $m = R_{A,\pi} \circ R_{C,\pi/2}$
a) What is the image of the point B under this composition m?
b) Identify the rigid motion m.

Q 21 Let the square ABC have corners A at (0,0), B at (1,0), C at (1,1) and D at (0,1). Let m be the rigid motion which is the composition $\gamma_{AD} \circ \gamma_{CB}$.
a) Find the image of the point A under motion m. b) Identify the rigid motion m.

Q 22 Playfair's Postulate: for every line l and every point P not on l, there is a unique line m parallel to l containing P.
The Euclidean Parallel Postulate: If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.
Prove that in neutral geometry, the Euclidean parallel postulate implies Playfair's postulate. That is, assume Euclid's Parallel Postulate and prove Playfair's Postulate.

Q 23 Find the image of the circle of radius 2 centered at the origin under inversion through the circle of radius 5 centered at (3,0).